One-dimensional Bose gases

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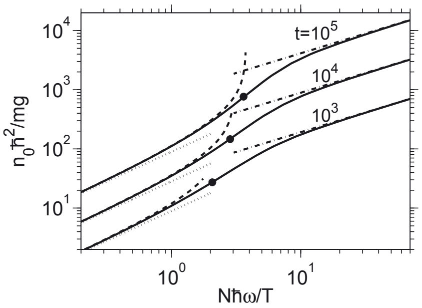
The one-dimensional (1D) Bose-gas model of particles interacting via a delta-function potential is of fundamental importance to quantum many-body physics. The model is exactly solvable for arbitrary interactions and temperatures, and it is now experimentally realizable with ultra-cold alkali atoms in highly anisotropic trapping potentials. This means there are unique opportunities for accurate tests of theory that were previously unavailable. In 2006 we have made further progress in our highly successful stream of theoretical studies of 1D Bose gases [1].

1. We have studied the nature of the transition to a coherent state in a harmonically trapped repulsive 1D Bose gas in the weakly interacting regime [2]. We have found a parameter space where the transition can be identified as an *interaction-induced* crossover to a quasi-condensate, which is present in both a finite-size system and in the thermodynamic limit. We predict that this scenario occurs at the crossover atom number $N_{\rm co} = (T/3\hbar\omega)\ln(\hbar^2 T/mg^2)$ or the crossover temperature $T_{\rm co} = 3N\hbar\omega/\ln(N\hbar^3\omega/mg^2)$. Here, ω is the trap frequency, $g > 0$ is the 1D coupling, N is the total atom number, and T is the temperature (in energy units). The interaction-induced crossover is contrasted to Bose-Einstein condensation in a harmonically trapped *ideal* gas, which occurs as a pure *finite-size* effect (absent in the thermodynamic limit) at a critical number $N_C = T/(\hbar\omega)\ln(2T/\hbar\omega)$.



The figure shows the peak density n_0 (in units of $mg/\hbar^2$) of a harmonically trapped 1D Bose gas versus $N\hbar\omega/T$ for three values of $t = 2\hbar^2 T/mg^2$ [2]. The three black dots show the crossover atom number $N_{\rm co}\hbar\omega/T$ for each t. The numerical results (solid line) obtained using the exact uniform solutions and the local density approximation are compared with the behavior in the quasi-condensate regime (dash-dotted lines), with the ideal Bose gas result (dashed lines), and the classical Boltzmann gas (dotted lines).

2. We have analyzed the thermodynamic properties of an array of independent 1D Bose gases in a 2D optical lattice. In particular, we have calculated the total entropy of the system and compared it with the respective result for the 3D BEC [3]. The objective is to analyze how the temperature of the system is altered upon an adiabatic (entropy preserving) transfer of the 3D gas into 1D tubes. Our results can be applied to the recent experimental measurements of the local pair correlation in Weiss's group [4], which can potentially include finite temperature effects with no fitting parameters. We propose that the 1D pair correlation measurements can be used as a new method of thermometry.

3. For the 1D gas with attractive interactions we have solved the Lieb-Liniger equations for the exact many-body wave functions for up to 20 particles. This system exhibits a symmetry-breaking quantum phase transition to a localised soliton [5], and the exact energy eigenvalues were previously unknown. This work is being prepared for publication, and in the future we intend to calculate the correlation functions and study the dynamics of a sweep across the critical point.

References

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