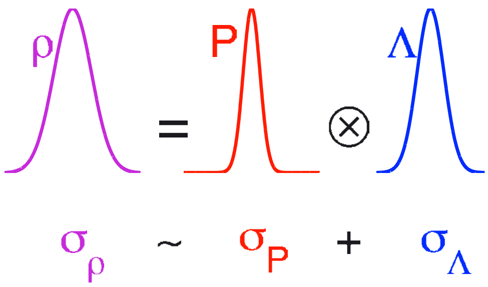
Phase-space representations for solving quantum many-body problems

D. W. Barry[1], K. K. Rajagopal[1], J. F. Corney[1], P. D. Drummond[1], S. Ghanbari[2] and T. Kieu[2]
[1]*ACQAO, School of Physical Sciences, The University of Queensland, QLD 4072, Australia*
[2]*ACQAO, Swinburne University of Technology, Melbourne, VIC 3122, Australia*

The physics of quantum many-body systems underlies the whole ACQAO research program, and the methods outlined here are central to the analysis of quantum correlations in ACQAO experiments. The goal of this research stream is to develop numerical techniques to solve quantum many-body problems at any given temperature. The methods are based on adapting and extending phase-space representations originally developed for quantum optical systems, to take into account the stronger interactions that occur in ultracold atoms.

The most fundamental way to generalise the previous phase-space methods is through the use of new operator bases, which determine the basic structure of the method and its suitability to different physical situations. We have developed the formalism for a generalised Gaussian basis for fermions. Besides generalising both the BCS ground states and noninteracting thermal states, which are both of physical relevance in ultracold Fermi gases, the Gaussian basis[1] provides a complete and positive representation of *any* physical fermionic state. Using these results, we have developed a class of quantum simulation techniques for Fermi systems[2].

$$\hat{\rho} = \int P(\vec{\lambda})\hat{\Lambda}(\vec{\lambda})d\vec{\lambda}$$



Left: Phase-space representations arise from an expansion P of the density matrix $\hat{\rho}$ over an overcomplete operator basis $\hat{\Lambda}$. **Right**: A good choice of basis can lead to a more efficient representation.

To obtain more precise calculations at low temperatures, the Gaussian techniques require the use of stochastic gauges. We have investigated the properties of the underlying nonlinear multiplicative SDEs, which are of higher-order than is usually encountered in other phase-space techniques. Based on this, we have developed and are now testing strategies that either minimize the tails that tend to develop at low temperatures, or, if the weights are allowed to be complex, to remove them totally[3, 4].

A more recent research program has been to apply these generalised phase-space methods to exact calculations of ultracold *bosons* in lattices. The topics of interest are quantum correlations in finite, trapped 1D gases (a UQ-Swinburne collaboration), and scalar and spinor BECs in rotating 2D lattices. In the bosonic case, the general Gaussian basis incorporates both coherent-state and thermal subsets, each of which provides a complete basis. There is thus a choice of strategies, from gauge-P augmented with number variables for onsite the on-site interactions, to the full Gaussian basis. To complement the phase-space study of the 2D lattice system, we are also applying and benchmarking phenomological approaches that treat vortices in such systems as a gas of interacting particles.

References

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