# Bose Condensates in Optical Lattices

## solid-state-like physics with cold atomic gases, and more

(Lecture # 2)

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# Atomic-Gas Bose-Einstein Condensates

- Many atoms (~$10^6$) in the same quantum state – internal state and center-of-mass motion

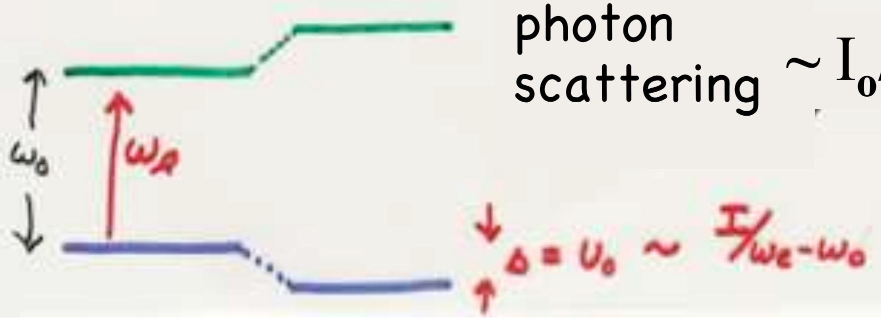
- Physical size ~100 mm – "macroscopic", many optical wavelengths

- Atom-atom interactions can be negligible or significant depending on circumstances and time scales.

# What is an optical lattice?

A: A periodic potential for atoms (in the gas phase), created by interfering laser beams

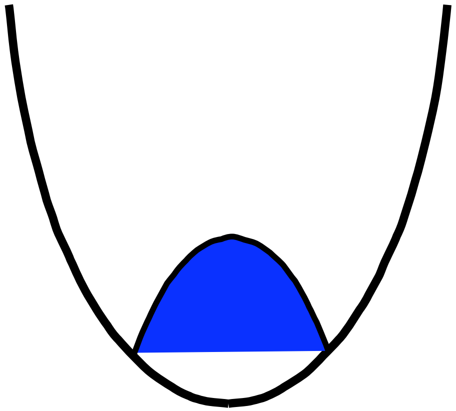

The Standing Wave created by counter-propagating traveling was, makes a periodic potential for atoms:

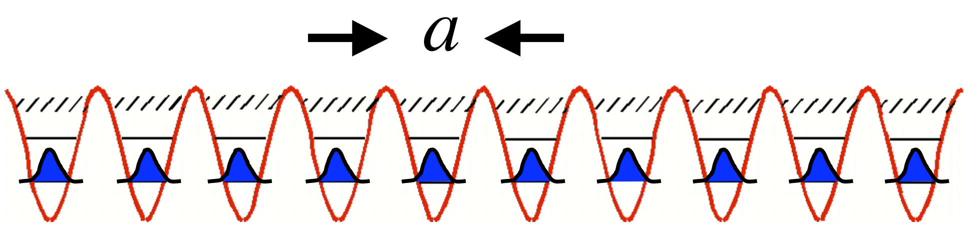

photon scattering $\sim I_o/(\omega_o-\omega)^2$

# A BEC in a optical lattice

Load a BEC, in a harmonic, magnetic trap…

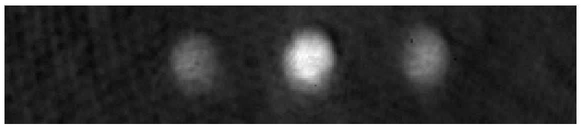


…into an optical lattice by "adiabatically" turning on the laser beams

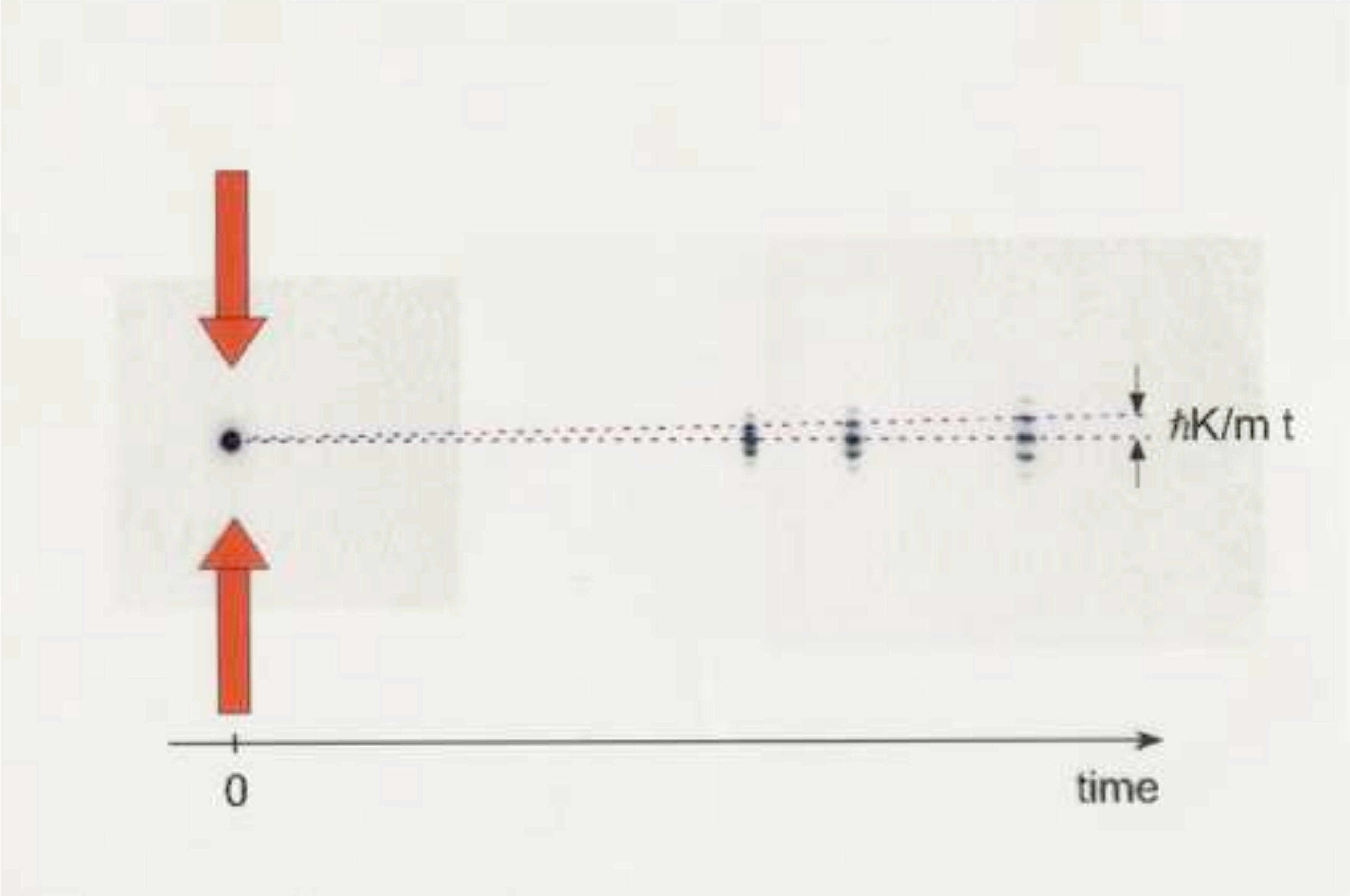



For non-interacting atoms this makes a mini-BEC in each potential well

Release non-adiabatically; after free-flight see momentum states--periodic wavefunction implies momentum components at multiples of twice the photon momentum ($2n\hbar k$)

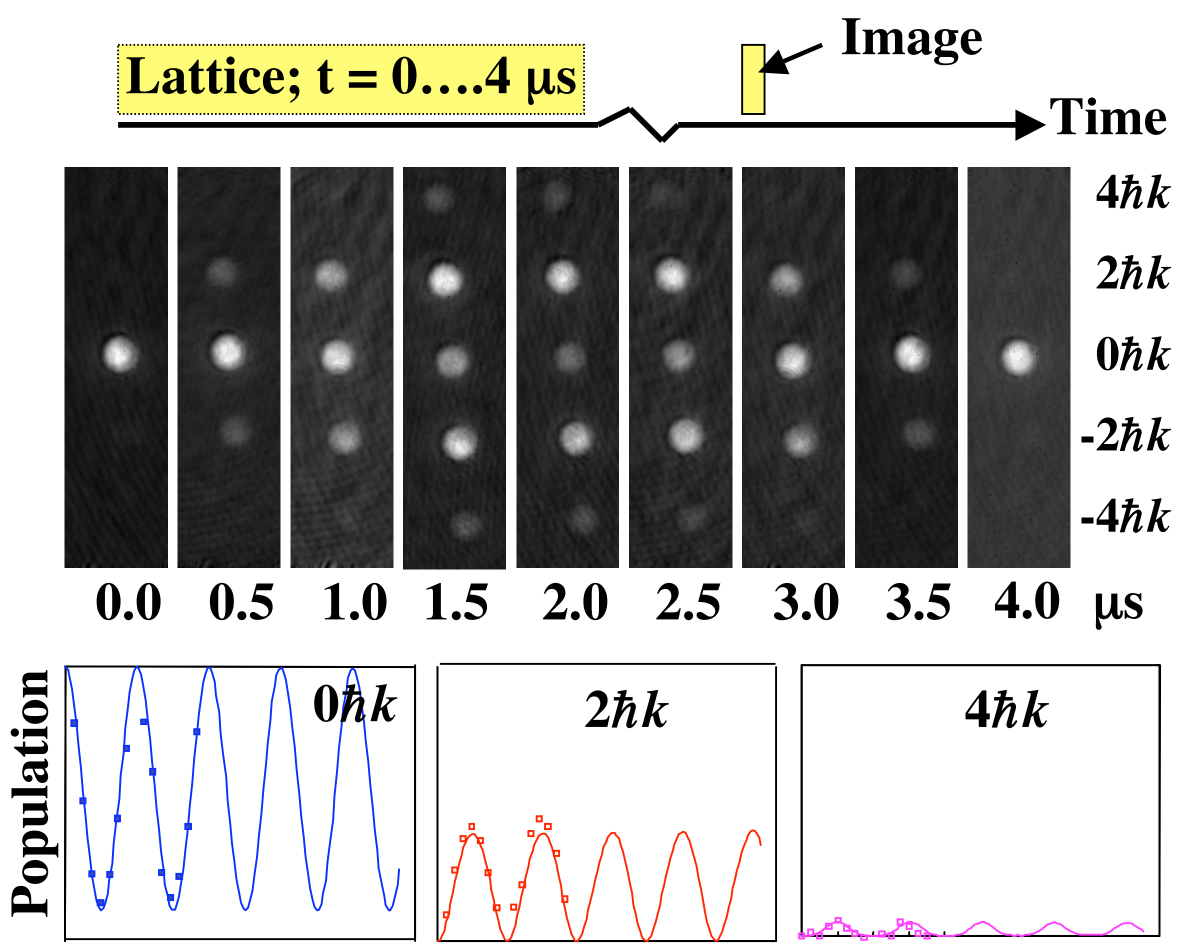


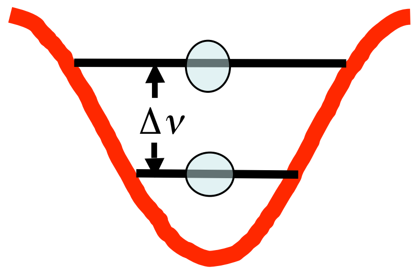
(This is the same as diffraction)



ℏK/m t
0
time

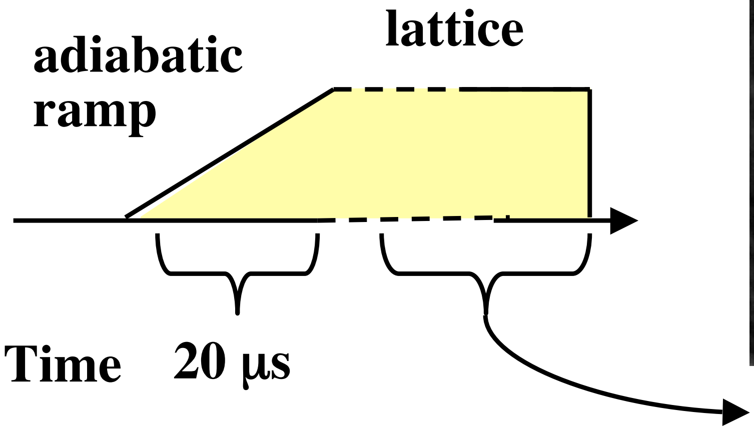
# Non-adiabatic Loading

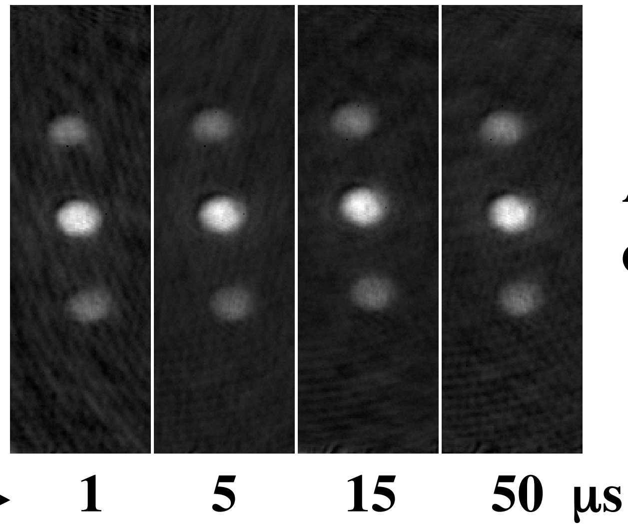



Sudden, non-adiabatic loading results in a superposition of motional states. The resulting beating is a signature of non-adiabaticity.

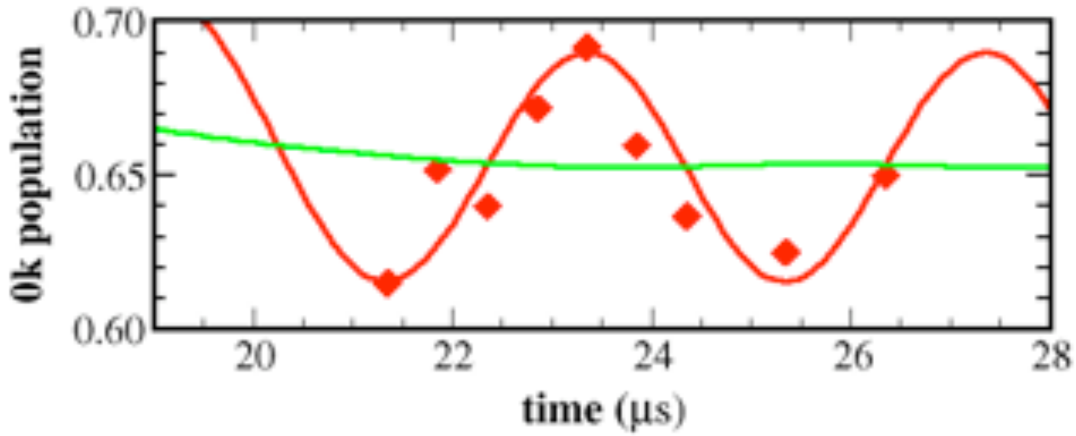
# Adiabatic (near perfect) Loading




**Are there still oscillations?**

**Population of $0\hbar k$ after "adiabatic" turn on**

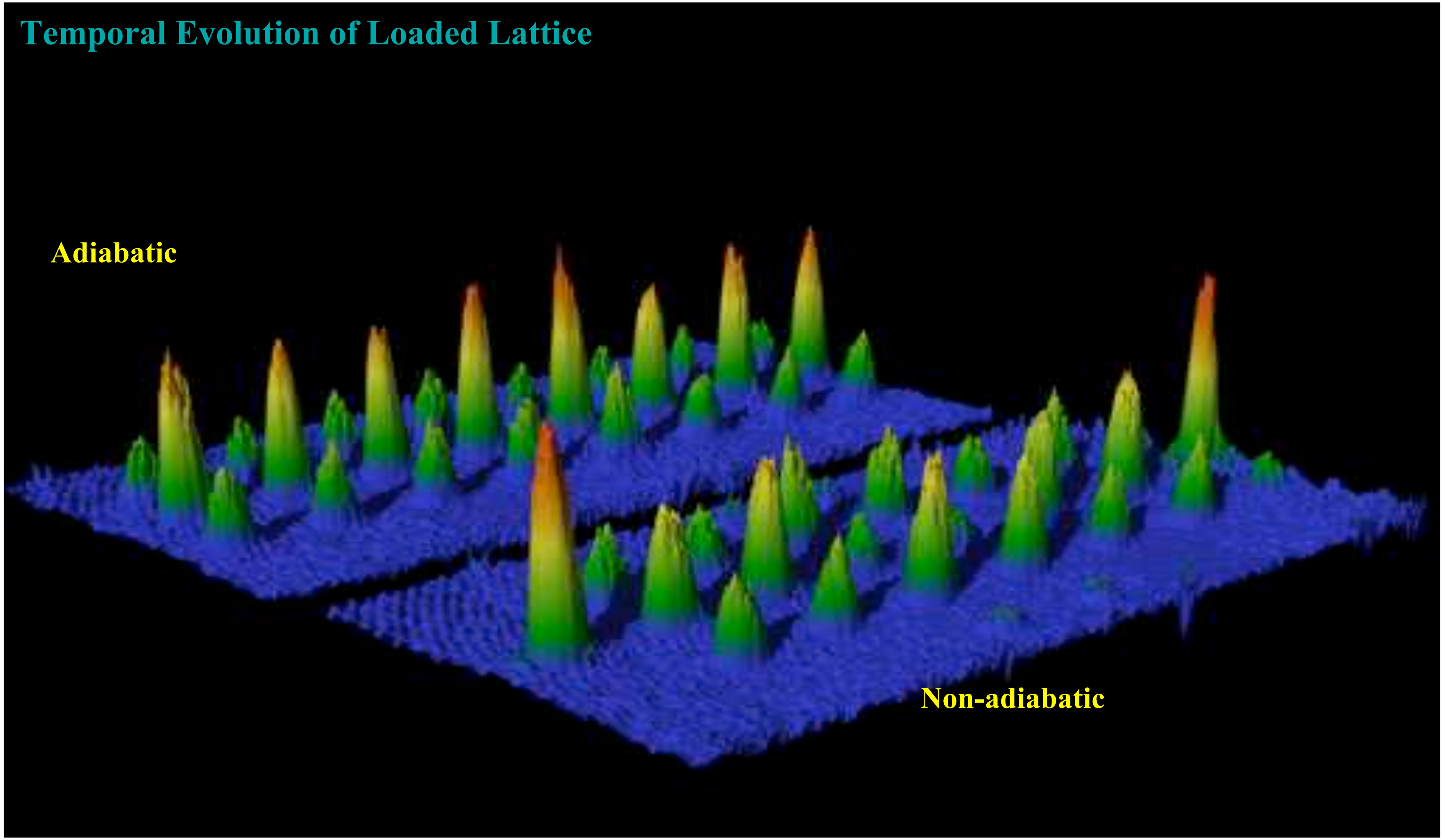



**Sensitive interference effect:**

**Expt shows 99.6% of population in lowest band**

**Theory shows how To get > 99.99%**

# NIST Experimental Data



Atoms moving in the periodic potential of an optical lattice are similar to electrons moving in a periodic crystal lattice.


But: lattice constant is 100s of nm, compared to Ås in crystals

No lattice defects

No phonons

Lattice potential is exactly known, spacing, strength, and geometry is variable

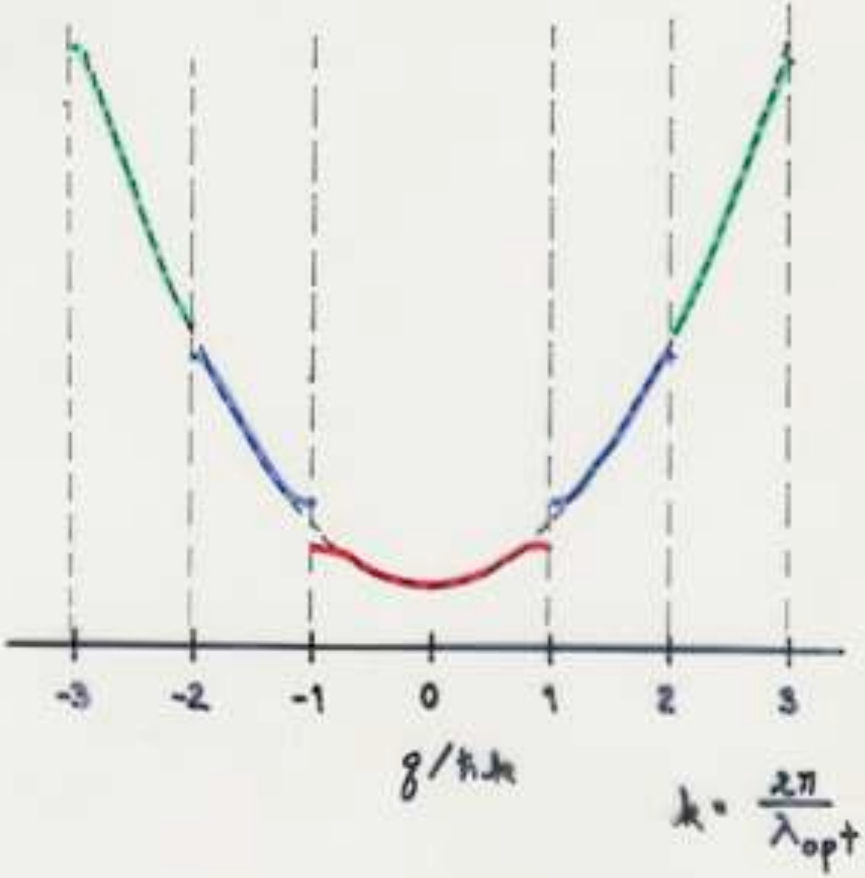
We use band structure and Bloch functions, familiar from solid state, to treat our system.

Bloch state: $\psi_{n,q}(x) = u_{n,q}(x)\, e^{iqx/\hbar}$

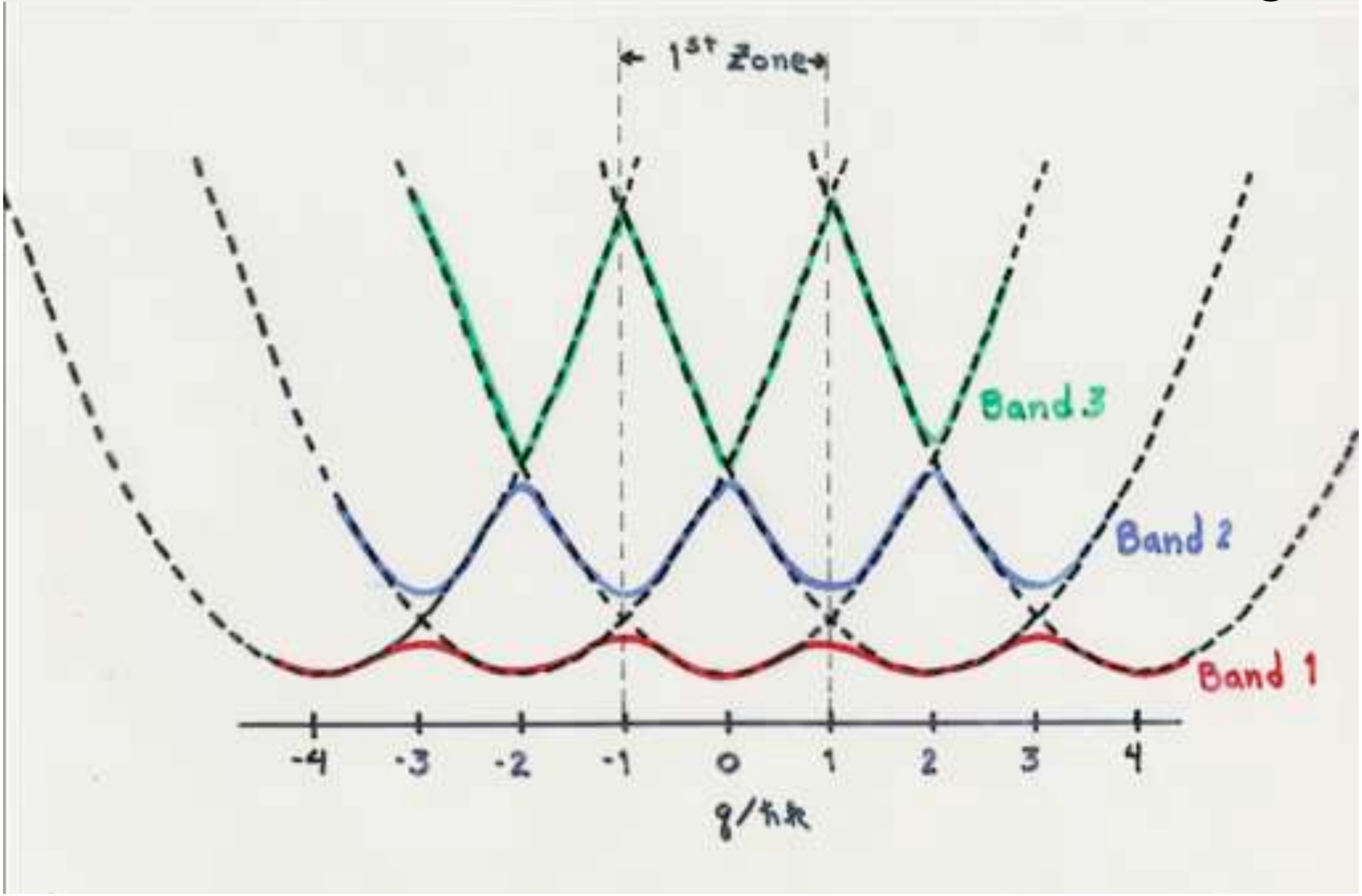
where $u_{n,q}(x+a) = u_{n,q}(x)$

i.e. $\psi(x)$ is periodic except for a phase $e^{iqa/\hbar}$. $q \equiv$ quasimomentum

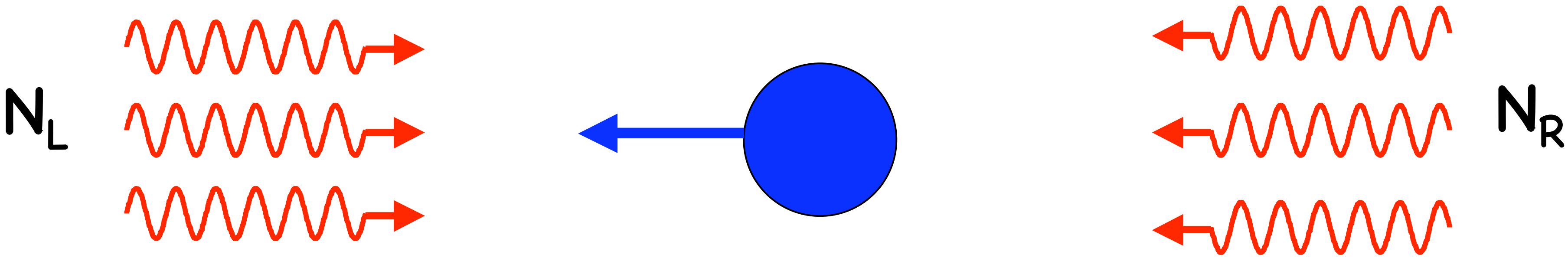
$q$ is modulo $\hbar K = h/a$, the reciprocal lattice momentum



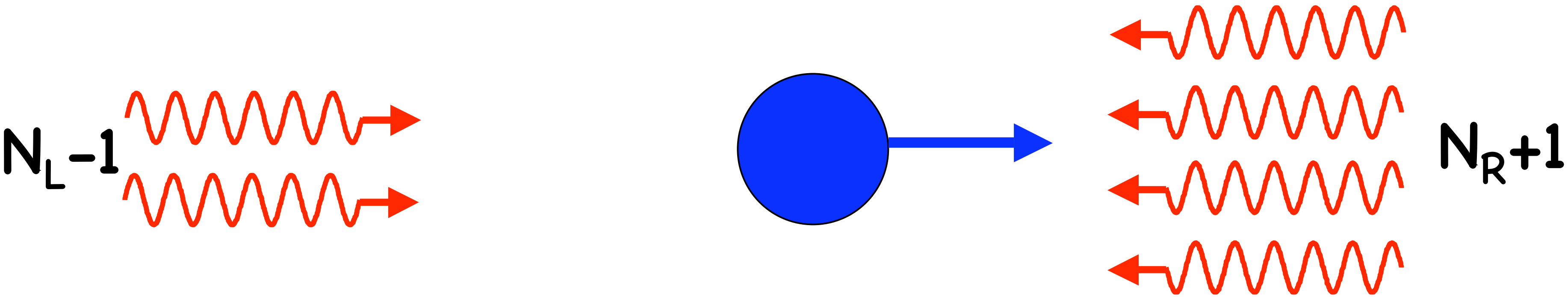
# Periodic Zone Scheme; note anticrossings



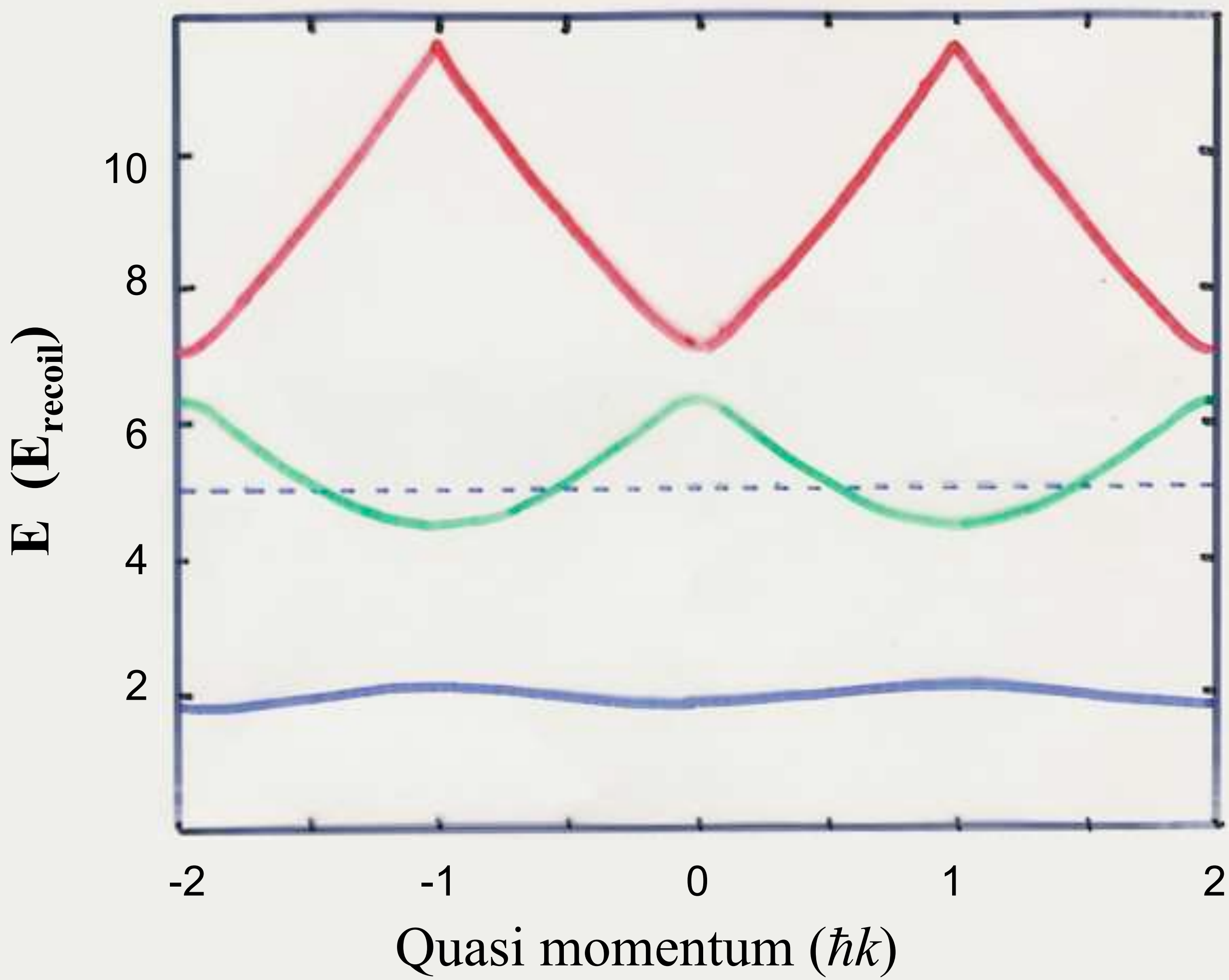
## The level (anti) crossings are at the points of Bragg diffraction, a degeneracy:



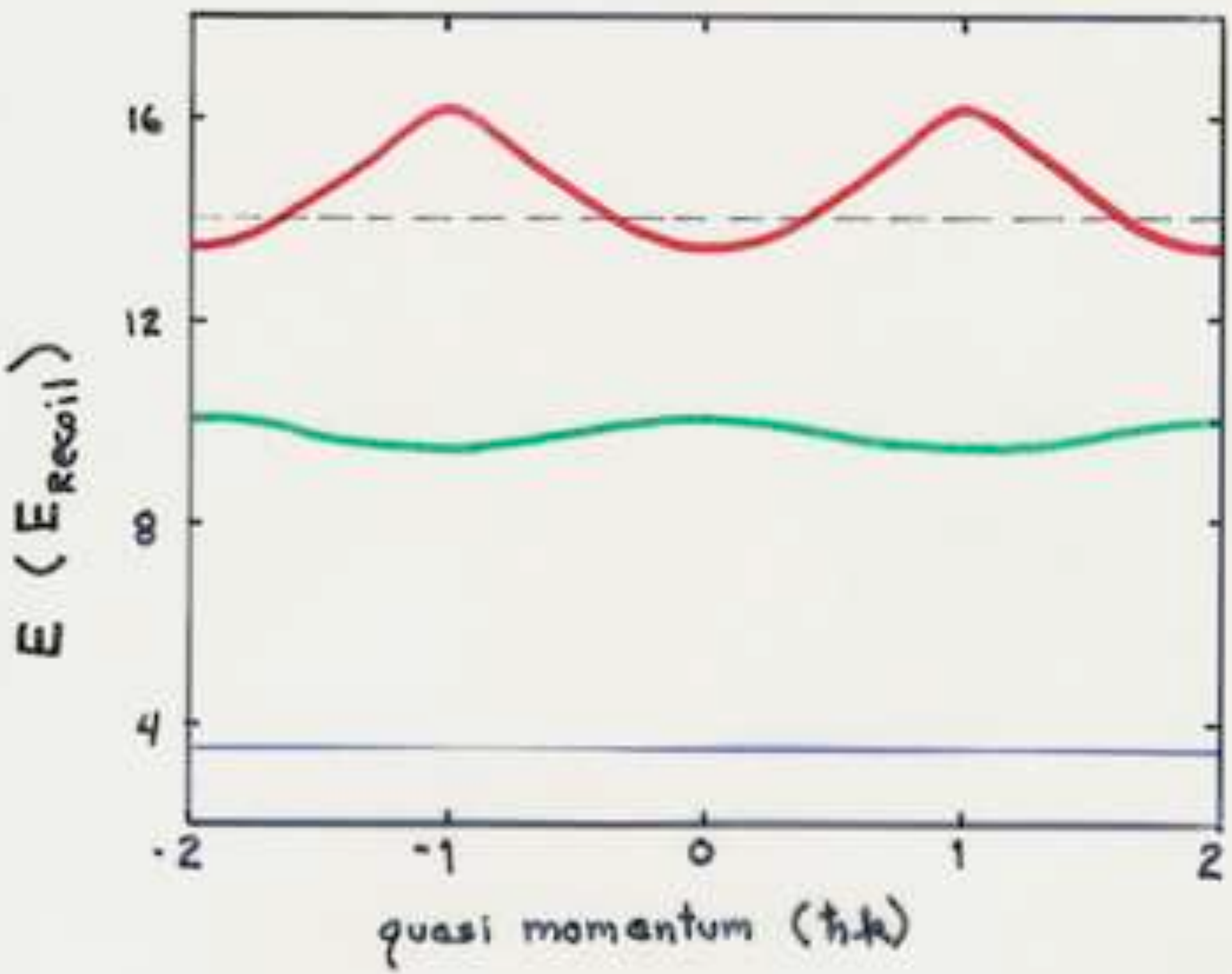

$$v = v_{\mathrm{rec}} = \hbar k/m$$



# Band Structure for 5 $E_R$ Lattice Depth



# Band Stucture for 14 $E_R$ lattice depth



Counter-propagating waves with different frequencies


make a <u>moving</u> periodic potential :

$$v_\ell = q/m = \frac{\Delta\nu}{2k}$$

A BEC, which has atoms with $\vec{p} = 0$ (constant phase across the BEC) looks like a <u>single</u> Bloch state $q = m v_{lattice}$ in the frame of the moving lattice.

# Motion of a BEC in a lattice

What happens to a BEC with a "single" $q$ depends on the dispersion relation $E(q)$, and specifically on $v_g$, the group velocity

$$v_{group} = dE(q)/dq$$

(in the *lattice* frame, so that $v_{lab} = v_g + v_{lattice}$ )

Free particle: $p = q$, $E(q) = q^2/2m$, $v_g = q/m$

A weak lattice is like free space, so $v_g = q/m = - v_{lattice}$ and $v_{lab} = 0$.

In a deep lattice $E(q)$ is flat, so $v_g = 0$, and $v_{lab} = v_{lattice}$,
Which means that the atoms are dragged along with the lattice.

Strong, moving lattice

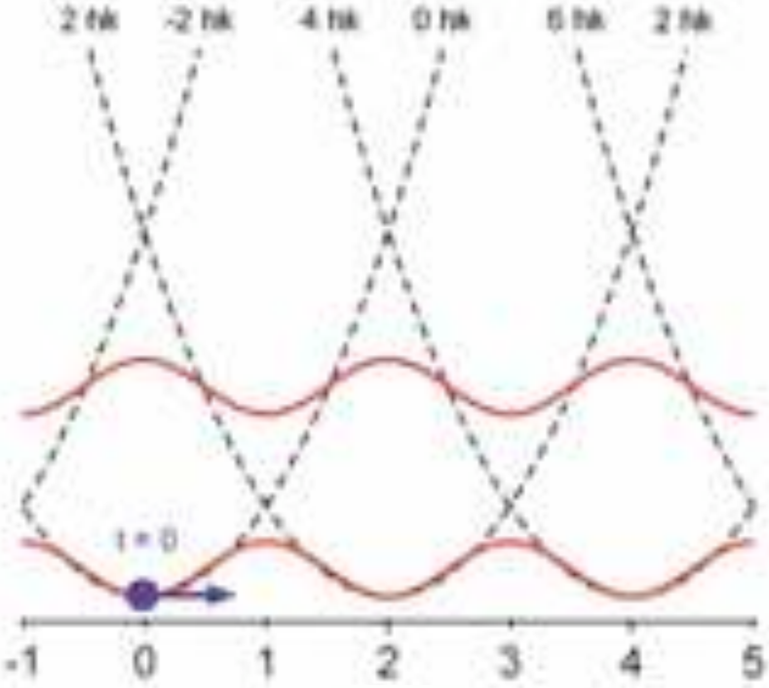

eigen state of the (individual) potential in moving frame ⇒ no phase gradient in moving frame ⇒ moves at $v_\ell$ in lab frame (but there is a well-to-well $\varphi$.

⟺

Weak, moving lattice


not an eigenstate of any individual potential well — tunneling is rapid, about as fast as $v_\ell$, so there is a phase gradient in moving frame; atoms at rest in lab.

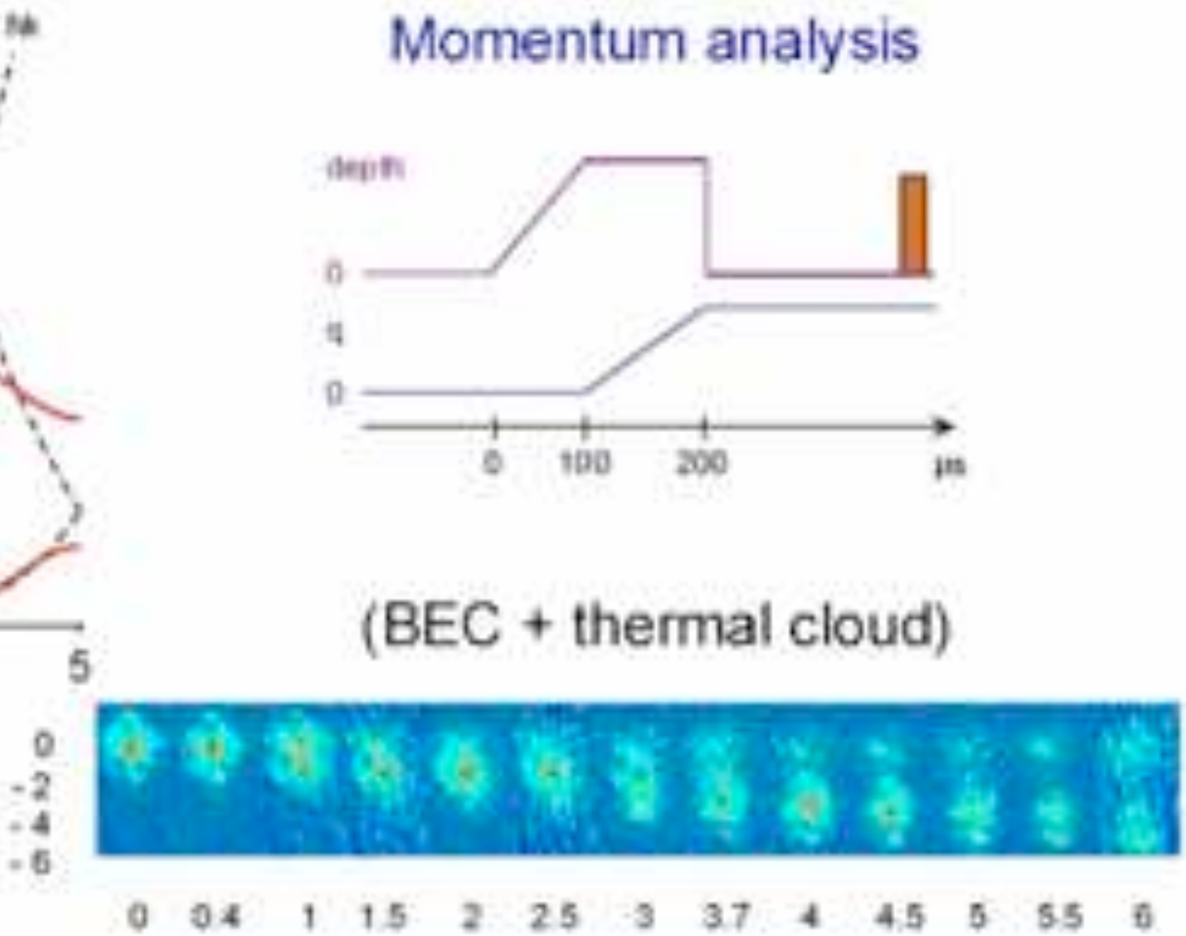
# Accelerate lattice, then analyze momentum by rapid turn-of



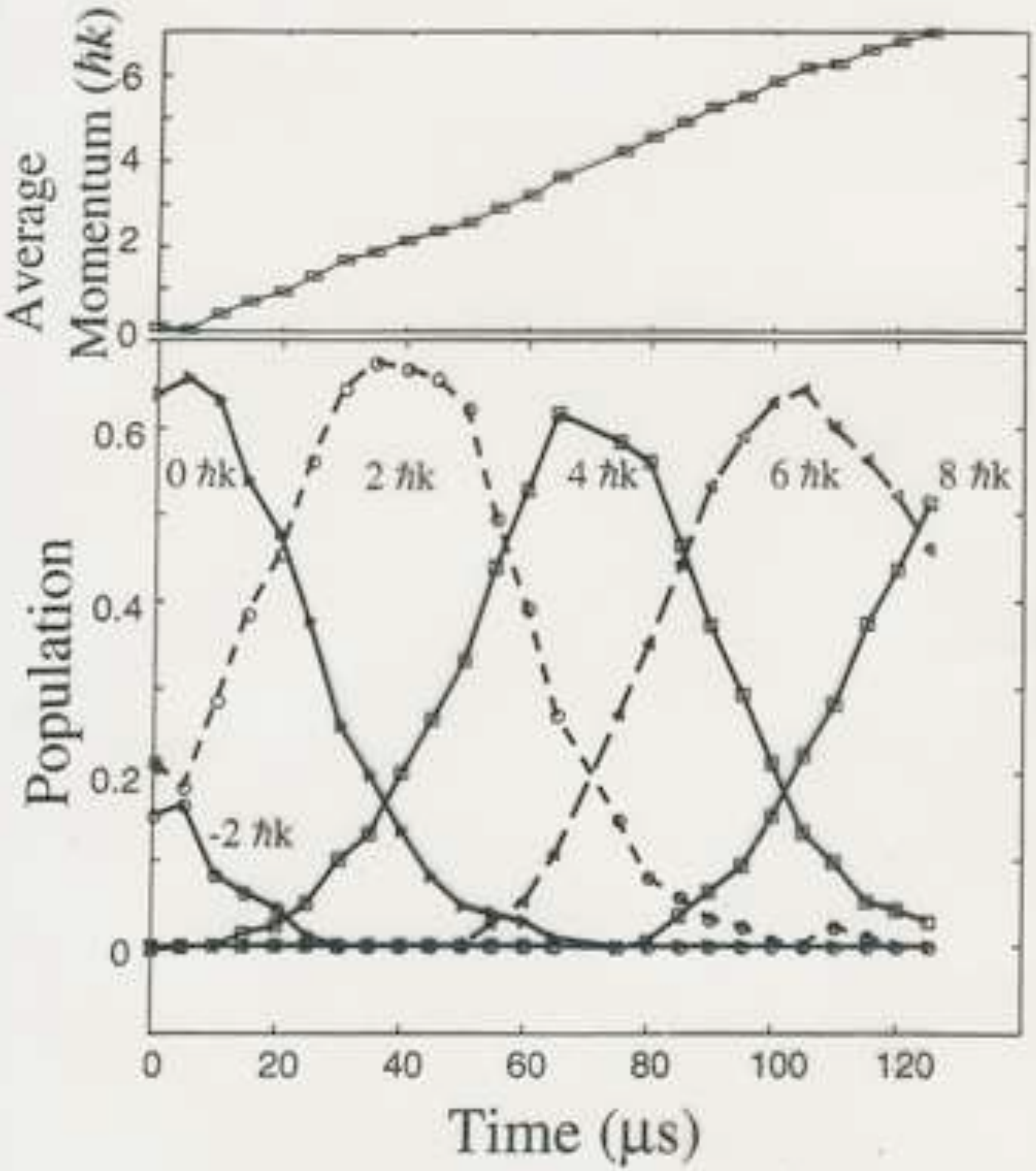

$q/\hbar k$

$\dot{q} = -m a$

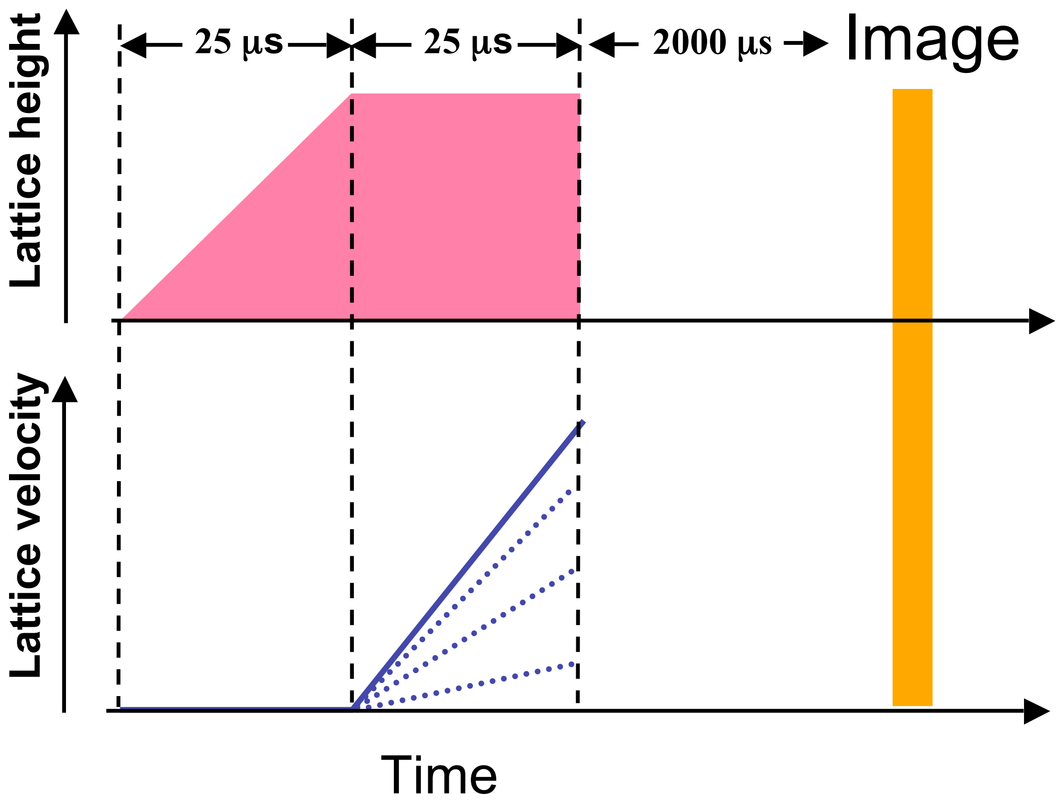
(related work in Pisa)

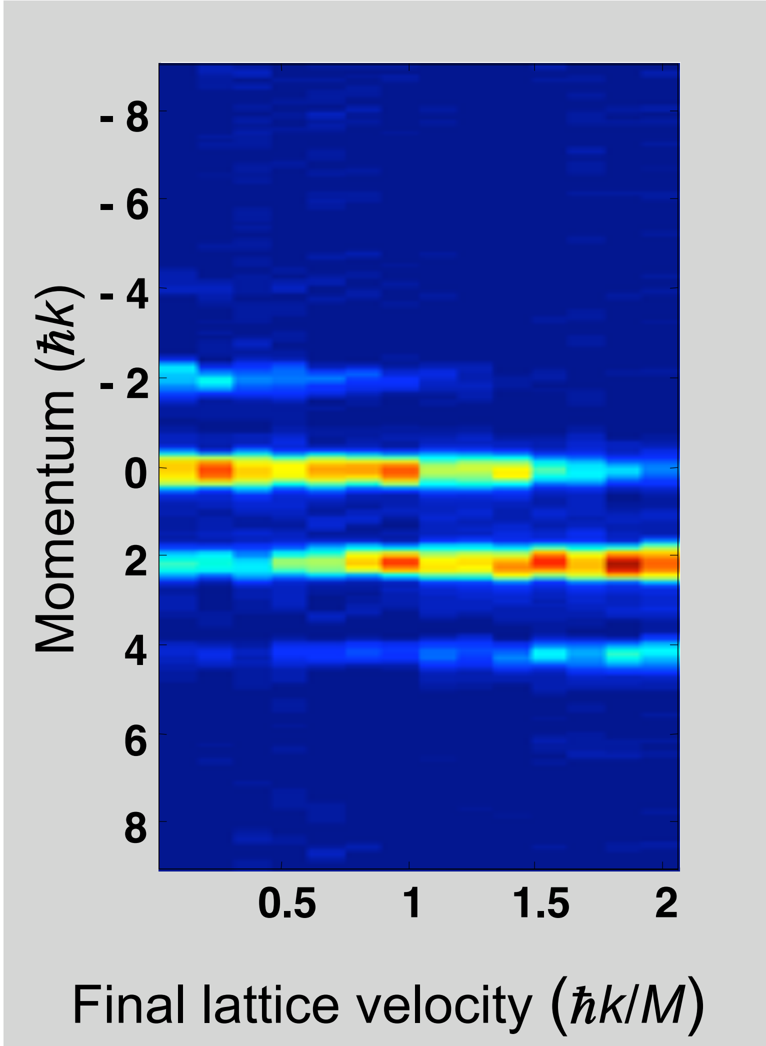



$q/\hbar k$

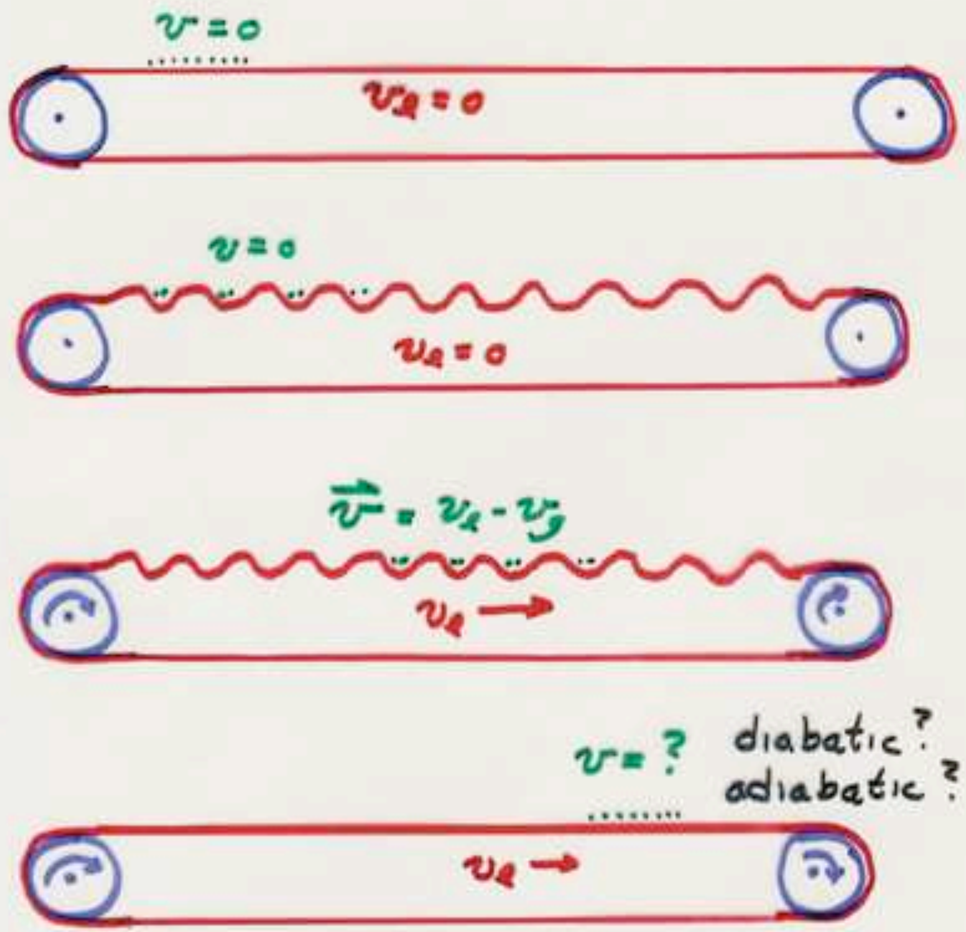

Average Momentum (ħk)
0
2
4
6
Population
0
0.2
0.4
0.6
0 ħk
2 ħk
4 ħk
6 ħk
8 ħk
-2 ħk
0
20
40
60
80
100
120
Time (µs)

# Different accelerations for constant time

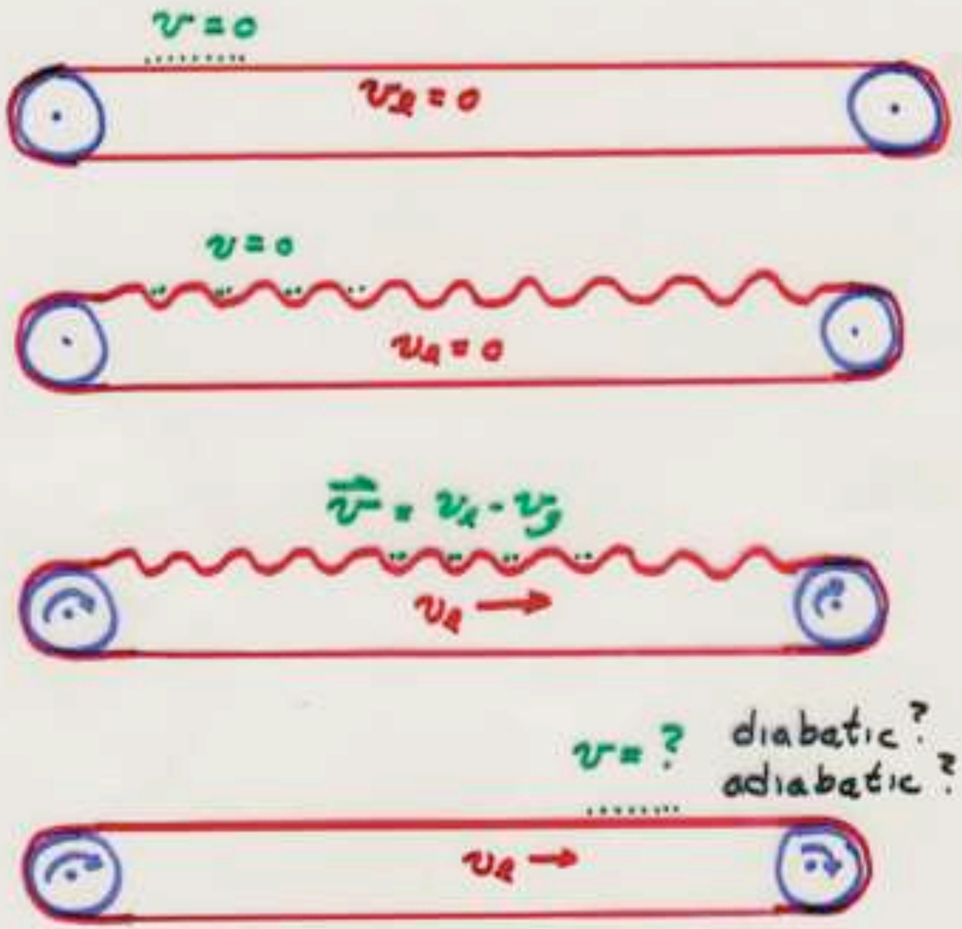


# A Fable about atoms accellerated by an optical lattice



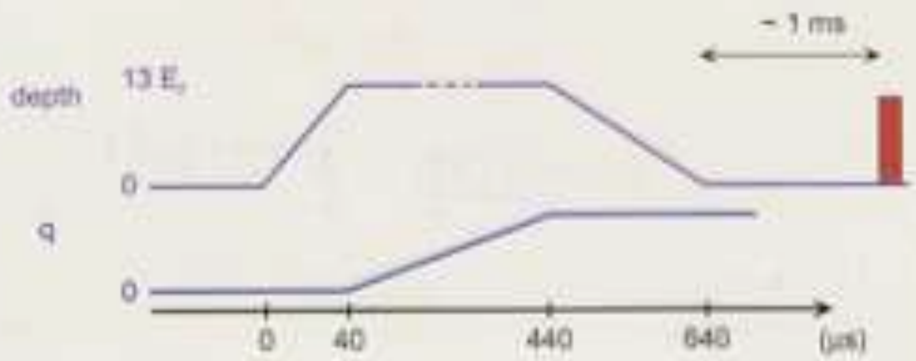
# A Fable about atoms accellerated by an optical lattice



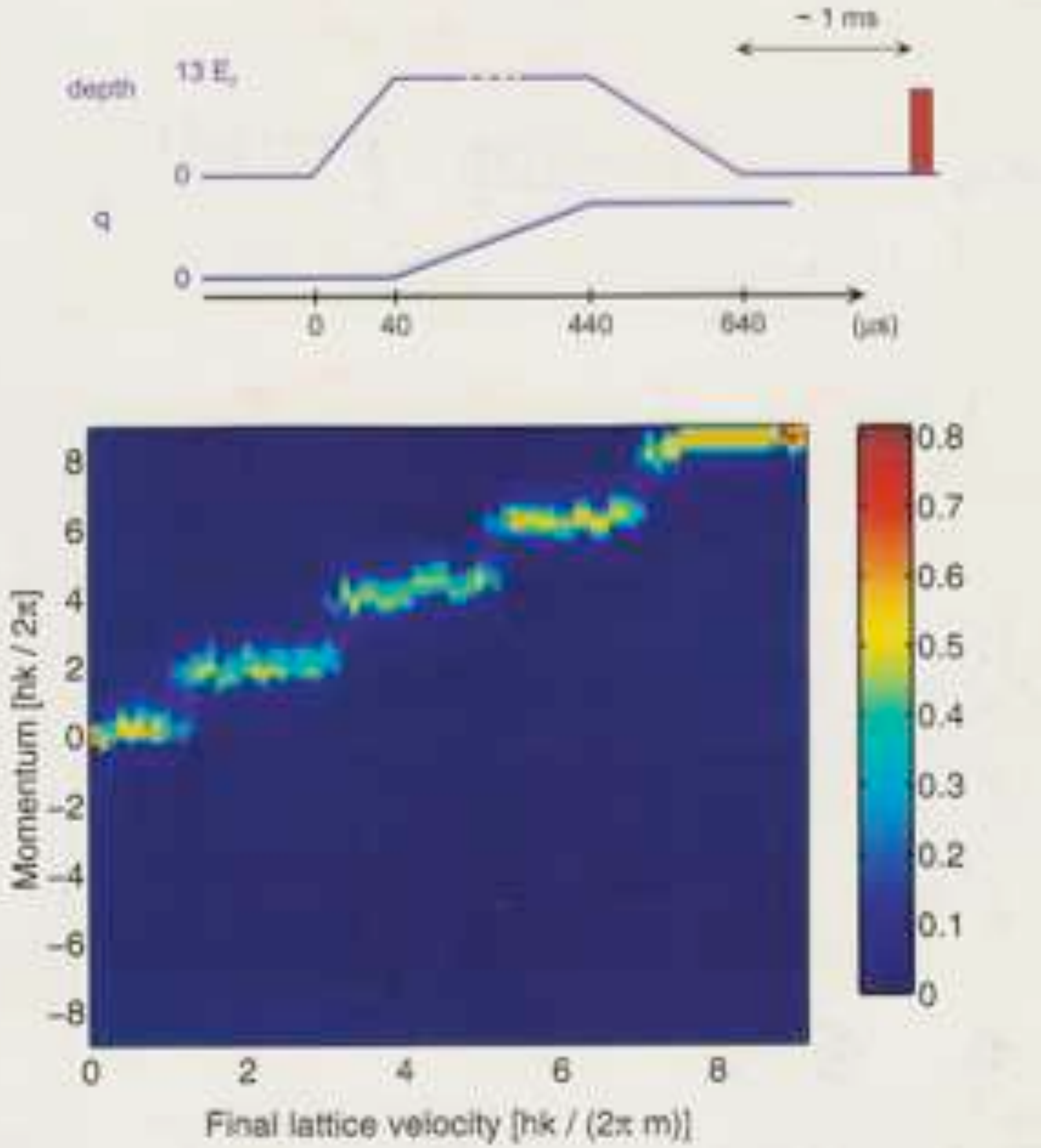

For a deep lattice and a fast (slow) turn-off, the velocity will be:

a. $v_{lattice}$
b. zero
c. something between
d. don't know
e. none of the above

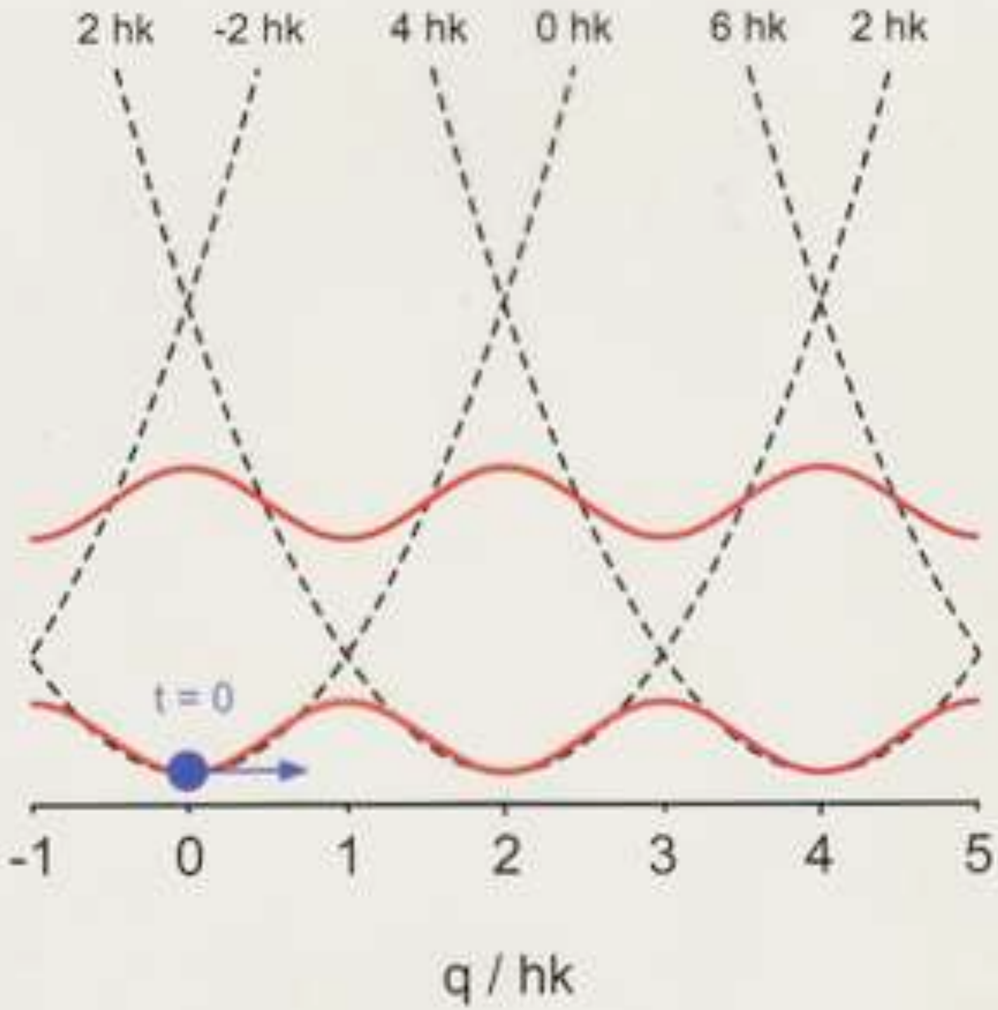
# BLOCH ACCELERATION IN THE GROUND STATE



# BLOCH ACCELERATION IN THE GROUND STATE



# BLOCH ACCELERATION IN THE GROUND STATE



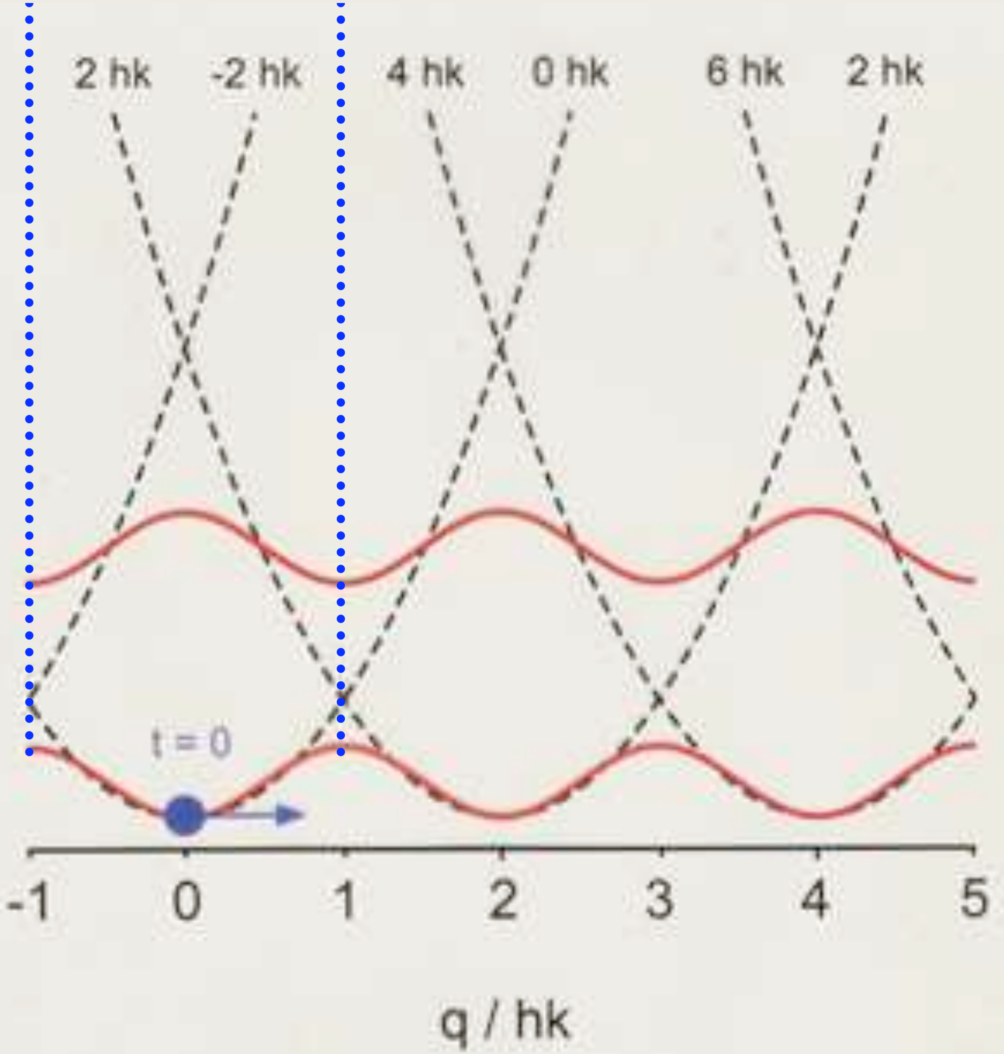
The “steps” in momentum are due to Bragg scattering at the Brillouin zone boundary.

Viewed in the lattice frame, these are, effectively, Bloch oscillations – a phenomenon that “freezes” accelerated particles in a periodic potential.

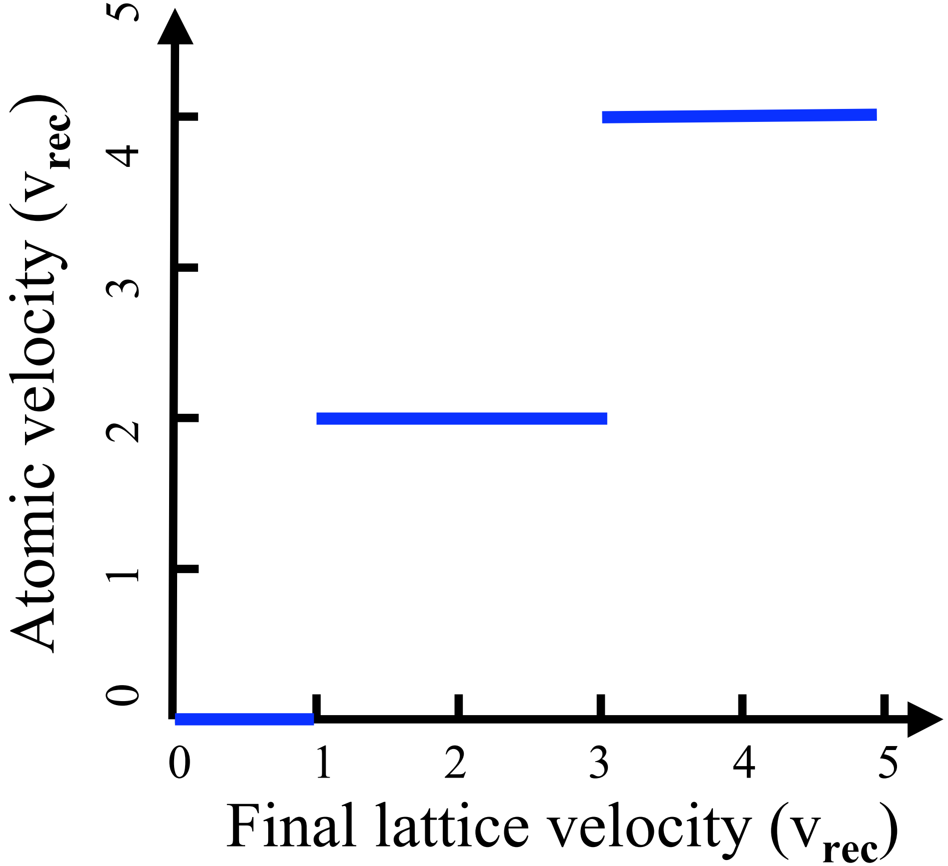
Bloch oscillations of electrons in crystals are very hard to see.

In optical lattices, they are routine.

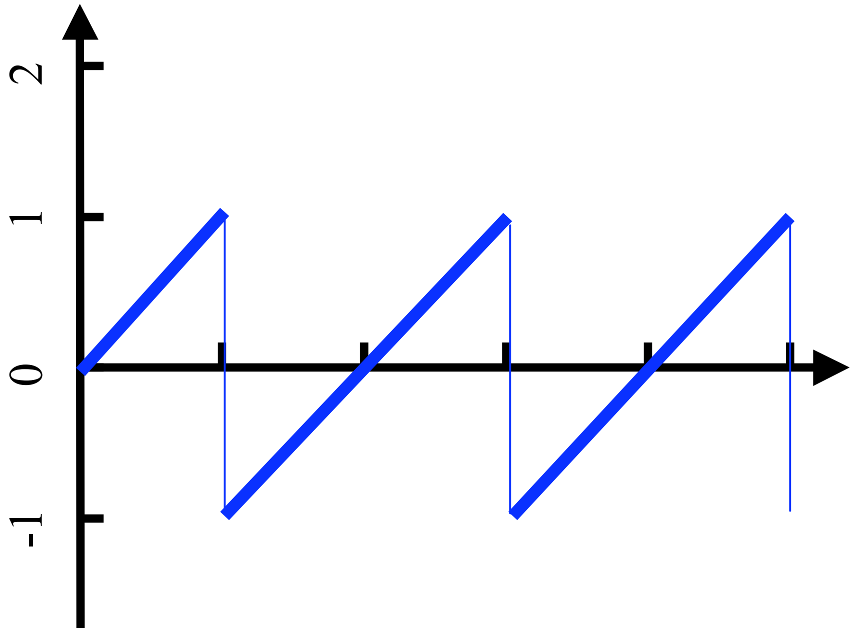
**In the folded zone scheme (staying in the lowest band) an accelerated atom is Bragg-reflected at the zone boundary, appearing at the opposite zone boundary, its velocity oscillating between the extremes of the group velocity in the band.**



**Acceleration followed by adiabatic turn-off is like seeing Bloch oscillations in a vanishingly shallow lattice (since the lattice is turned down to zero), so the velocity oscillations are from - $v_{rec}$ to + $v_{rec}$.**

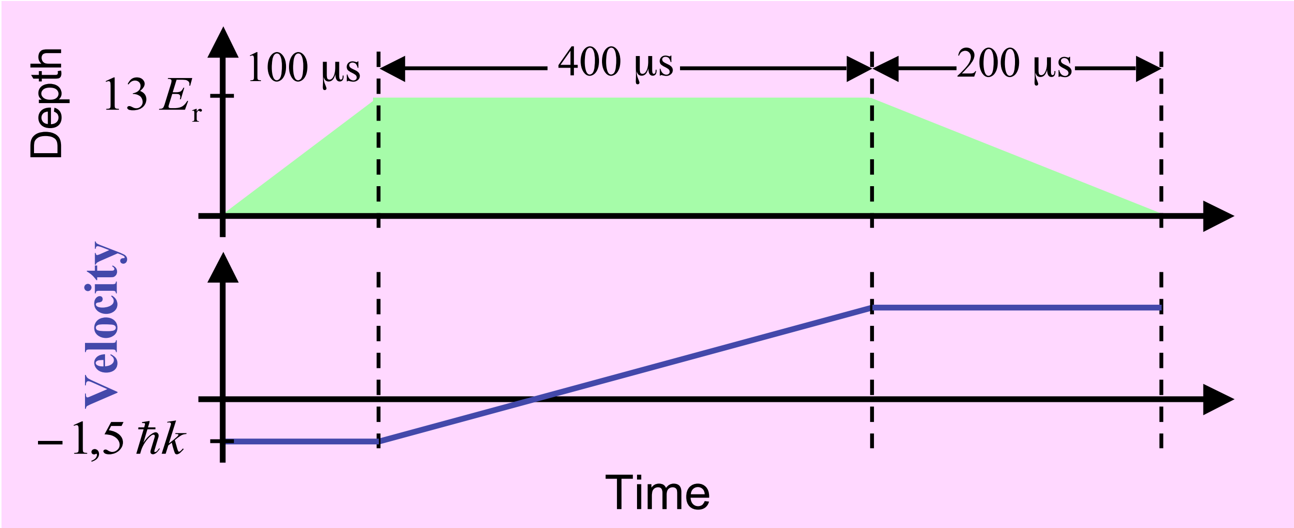



**LAB FRAME**
**(steps)**

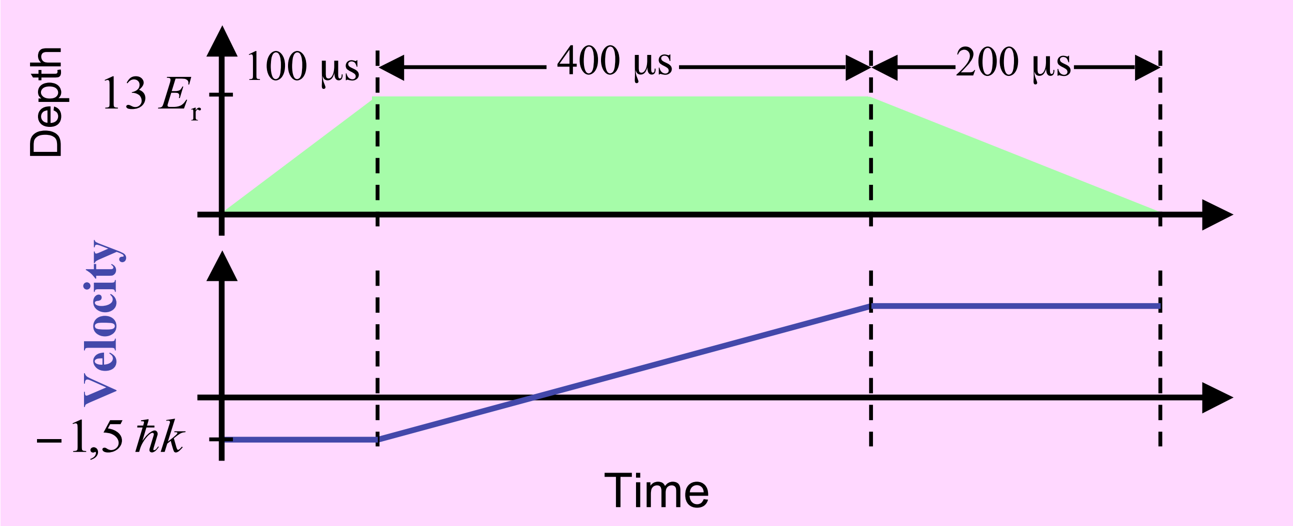


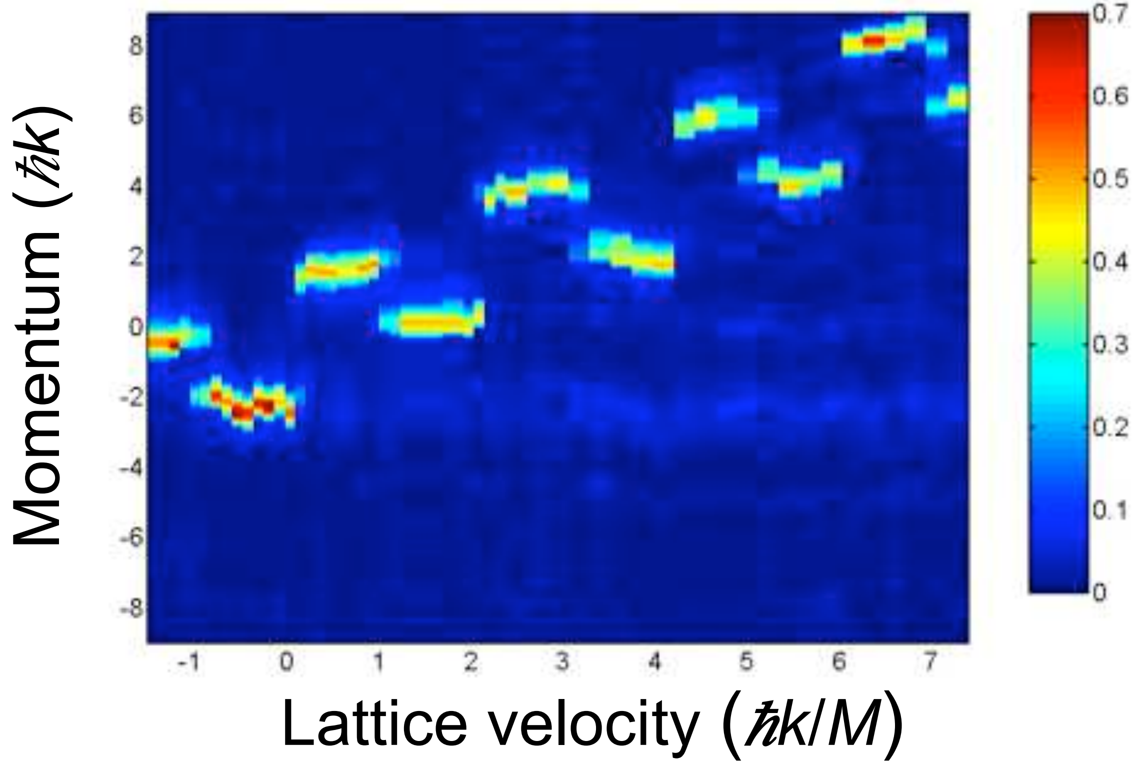

**LATTICE FRAME**
**(Bloch oscillations)**

# Bloch Acceleration in the Second Band

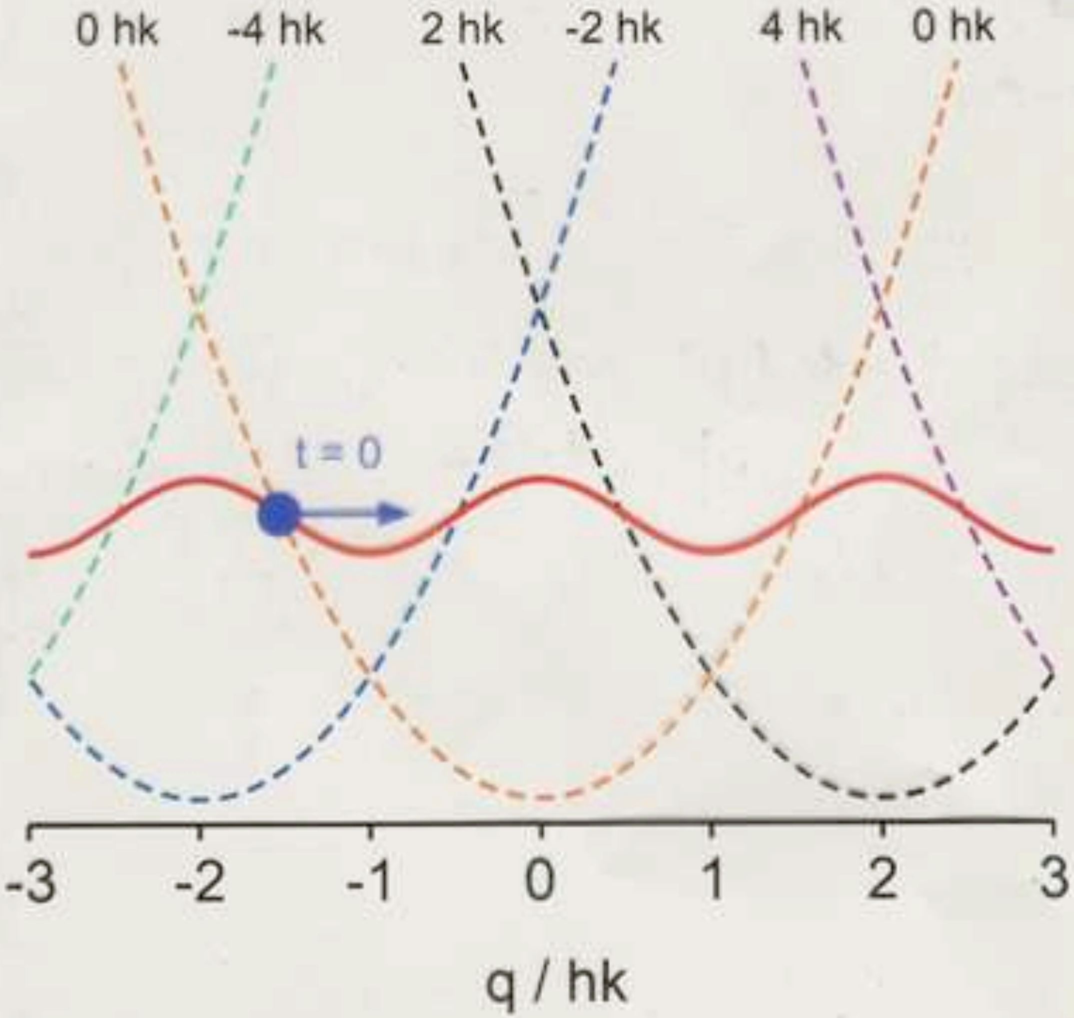


# Bloch Acceleration in the Second Band



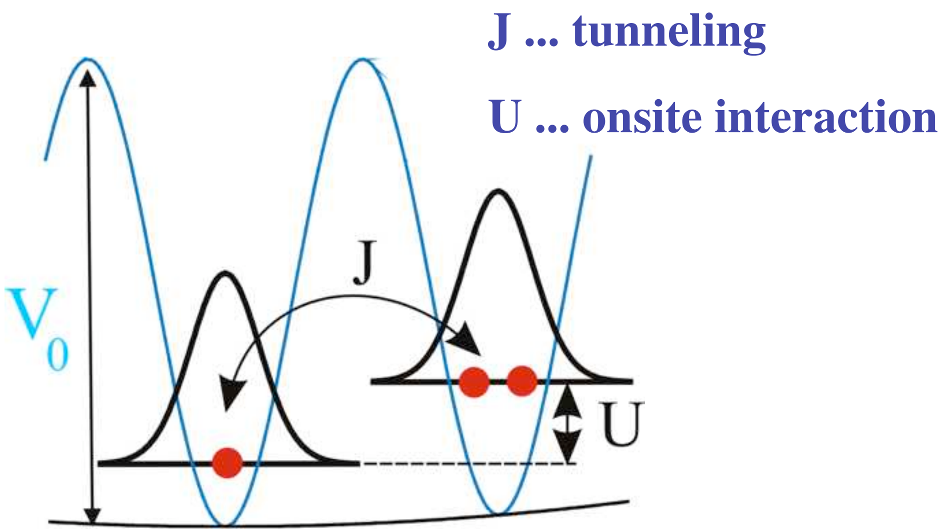
# BLOCH ACCELERATION IN THE SECOND BAND



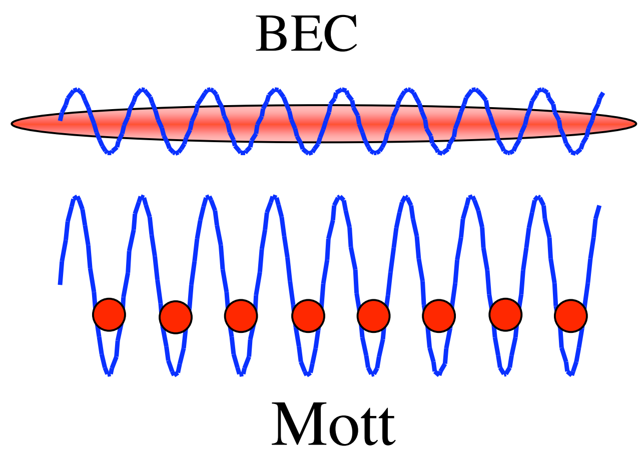
All of these experiments are “single-atom” experiments – they do not depend on the interactions!

The Mott insulator transition <u>uses</u> the interactions to make a fundamental change in the way the atoms are arranged in the optical lattice.

# The Mott transition: BEC goes to Fock state



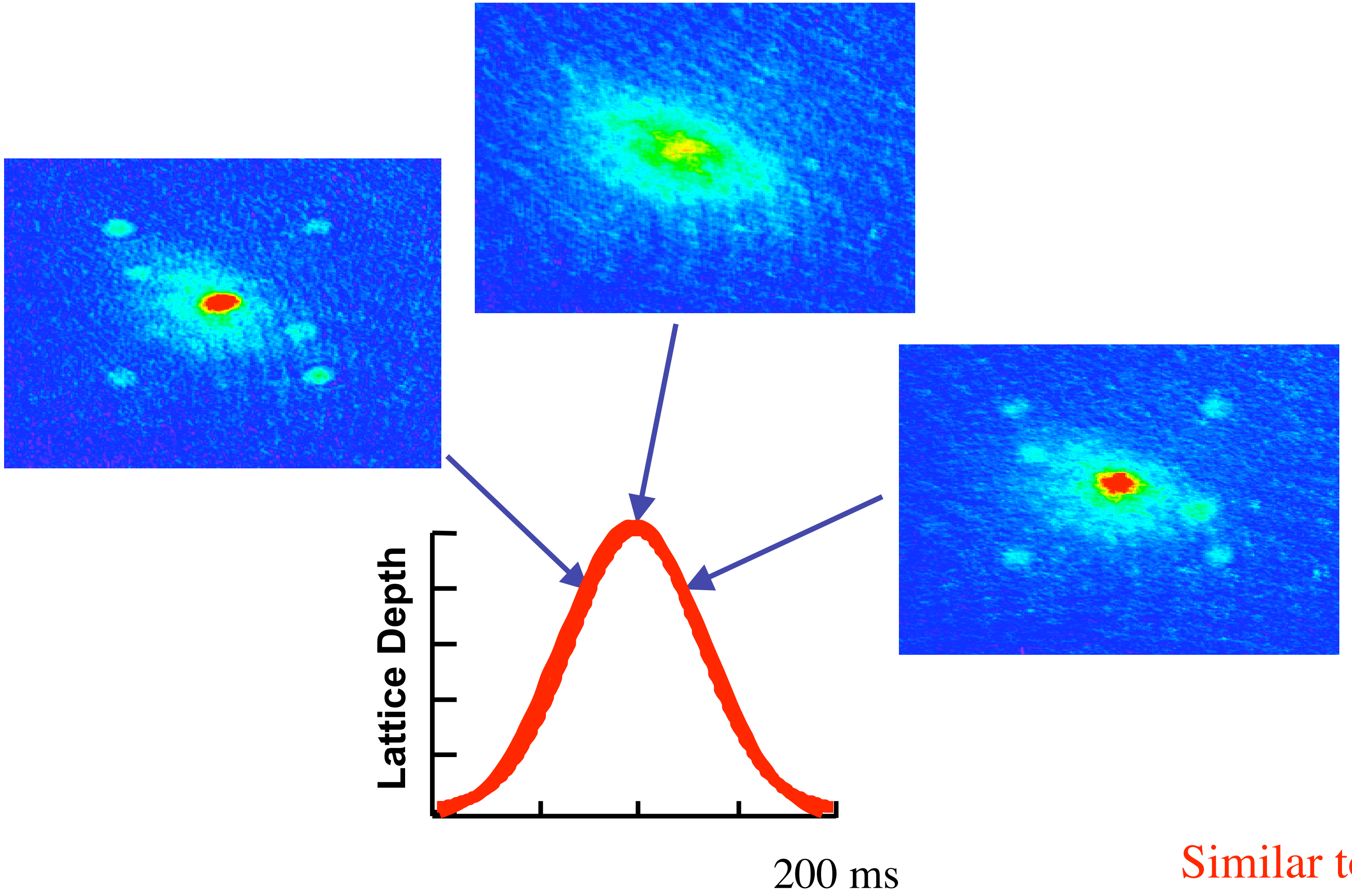

Courtesy of Peter Zoller




When U is large compared to J, the ground state has one (or some other integer) atom per lattice site.

According to theory, ground state provides near-perfect filling of one atom/lattice site: At $V_0 = 35\ E_R$, < 5% chance of *any* of $10^5$ sites having an error.

# Mott transition seen as disappearance of 3D diffraction pattern




*Phil. Trans. R. Soc. Lond.* A **361**, 1417 (2003)

Similar to Greiner et al. Nature **415**, 39, (2002).

# THE END

(of Phillips lecture # 2, on BEC in optical lattices)